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THE STRENGTH OF I-BEAMS IN FLEXURE

BY

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ENGINEERING EXPERIMENT STATION

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ENGINEERING EXPERIMENT STATION

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THE STRENGTH OF I-BEAMS IN FLEXURE

By Herbert F. Moore, Assistant Professor in Theoretical and Applied Mechanics

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THE STRENGTH OF I-BEAMS IN FLEXURE.

1. *Introduction.*—The mathematical theory of the resistance of materials in flexure has been extensively developed, but much less has been done in the experimental study of the phenomena of flexural stress. A striking fact brought out in the tests which have been made is the tendency of metal beams to fail by reason of column action in fibers which are under compressive stress. This tendency is especially strong in I-beams, channel-beams, and other forms of beams having tension and compression flanges connected by a comparatively thin web. The tests of Marburg*, Christie†, and Burr and Elmore‡ show that this column action in I-beams may cause failure of test pieces by sidewise buckling or on account of excessive stresses in the web. These tests emphasize the importance of taking into account of stresses other than the direct flexure stresses in the flanges.

The wide-spread use of I-beams as flexural members makes the subject of their flexural strength a matter of general engineering interest. To obtain experimental data on the action of I-beams under load, several series of tests of I-beams were carried out in the Laboratory of Applied Mechanics of the University of Illinois. This bulletin records and discusses the results of these tests and of others of similar kind. A formula is deduced for the flexural strength of I-beams which are not restrained against sidewise buckling. There also is given a discussion of the stiffness of I-beams, a discussion of the action of I-beams restrained against sidewise buckling and restrained against twisting at the ends of the beams, and a discussion of web failure of I-beams.

2. *Acknowledgment.*—The experimental work was a part of the research work of the department of Theoretical and Applied Mechanics, and was done under the general direction of the head of that department, Professor A. N. Talbot. The tests were made under the direct supervision of the writer. Acknowledgment is hereby made to the following students in civil engineering who assisted in making tests: F. J. Weston and W. E. Deuchler of the class of 1910, and M. H. Froelich and F. C. Lohman of the class of 1911. Analytical methods devised by various investigators have been used in this bulletin, and experimental data from various sources have been quoted; in all cases an attempt has been made to give due credit for methods and data.

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*Proceedings of the American Society for Testing Materials, Vol. IX (1909), p. 378.

†Pencoyd Steel Handbook (1898 Edition), p. 23.

‡Selected Papers of the Rensselaer Society of Engineers, Vol. 1, No. 1. An abstract of the results of these tests is given in Burr's "Elasticity and Resistance of the Materials of Engineering," p. 694.

3. *Phenomena of Flexural Failure.*—The formulas commonly used for computing the stresses and deflections in beams are based on the assumptions (1) that a plane cross-section of a beam remains plane during flexure and (2) that the moduli of elasticity of the beam material for tension and for compression are equal and constant. For low stresses and for beams of medium or long spans these assumptions are very nearly exact. They become rough approximations when a beam is loaded to a point near failure.

The failure of beams of brittle material usually occurs by snapping of the extreme tension fibers at a computed fiber stress higher than the tensile strength of the material as determined by tension tests of specimens. Brittle material is nearly always much weaker in tension than in compression. As the fiber stress in beams of such material increases with increasing load, by reason of the change which takes place in the values of the moduli of elasticity, the tension side of the beam stretches more readily than the compression side shortens. The effect is to shift the neutral axis of the beam toward the compression side. This, together with the difference in strength, causes the actual tensile fiber stress to be less than the computed tensile fiber stress.

The failure of beams of ductile material may take place in one of a number of ways:

(1) The beam may fail by direct flexure. Under increasing load the usual flexure formulas are very nearly exact up to a load which stresses the extreme fibers of the beam to the yield-point strength of the material. When the yield point is reached in the extreme fibers, the deflection of the beam increases more rapidly with respect to an increase of load; and if the beam is of a thick, stocky section or is firmly held so that it can not twist or buckle, failure takes place by a gradual sagging which finally becomes so great that the usefulness of the beam as a supporting member is destroyed.

(2) In a beam of long span, the compression fibers act somewhat as do the compression fibers of a column, and failure may take place by buckling. Buckling failure, in general occurs in a sidewise direction. Sidewise buckling may be either the primary or the secondary cause of failure. In a beam in which excessive flexural stress is the primary cause of failure and in which the beam is not firmly held against sidewise buckling, the primary overstress may be quickly followed by the collapse of the beam due to sidewise buckling. The sidewise resisting strength of a beam is greatly lessened if its extreme fibers are stressed to the yield point. Sidewise buckling may in some cases be a primary cause of beam failure, in which cases the computed fiber

stress, in general, does not reach the yield-point strength of the material. Sidewise buckling not infrequently limits the strength of narrow, deep beams, especially beams of I-section or channel-section with tension and compression flanges connected by a thin web. Whether it is a primary cause of failure or a final manner of failure, sidewise buckling results in a clearly marked and generally quite sudden failure of a beam.

(3) Failure in an I-beam or a channel-beam may occur by excessive shearing stress in the web, or by buckling of the web under the compressive stresses which always accompany shearing stress. If the shearing fiber stress in the web reaches a value as great as the yield-point strength of the material in shear, beam failure may be expected and the manner of failure will probably be by some secondary buckling or twisting action. The inclined compressive stress always accompaning shear may reach so high a value that the buckling of web of the beam is a primary cause of failure. Danger of web failure as a primary cause of beam failure exists, in general, only for short beams with thin webs.

(4) In the parts of beams adjacent to bearing blocks which transmit concentrated loads or reactions to beams, high compressive stresses may be set up, and in I-beams or channel-beams the local stress in that part of the web nearest a bearing block may become excessive. If this local stress exceeds the yield-point strength of the material at the junction of web and flange, the beam may fail primarily on account of the yielding of the overstressed part and finally by a resulting twisting action of the beam.

4. *Earlier Tests of I-beams.*—Data of a considerable number of tests of I-beams have been published. A list of some important tests with references follows:

(1) Tests of twenty 6-in. wrought-iron I-beams loaded at the midpoint of the span and so held in the testing machine as to be free to buckle sidewise. The spans varied from 4 ft. to 20 ft. These tests were made by Burr and Elmore at the Rensselaer Polytechnic Institute and are reported in "Selected Papers of the Rensselaer Society of Engineers," Vol. 1, No. 1. An abstract of the results is given in Burr's "Elasticity and Resistance of the Materials of Engineering," 1905 edition, p. 694.

(2) A series of tests on wrought-iron I-beams made by Tetmajer at the Materialpruefungsanstalt at Zurich, Switzerland. The results of the tests are given in Heft IV. of the Mitteilungen of that laboratory. The results of nine of the tests are given in Lanza's "Applied Mechanics," 1905 edition, p. 443.

(3) A series of tests of twenty-one steel I-beams made by Christie at the Pencoyd Iron Works. In these tests the beams were somewhat restrained from sidewise buckling by the friction of the bearing blocks which were directly attached to heads of the testing machine. These tests were reported by Mr. Christie in the Transactions of the American Society of Civil Engineers for 1884. A summary of results is given in Burr's "Elasticity and Resistance of Materials of Engineering," 1905 edition, p. 689.

(4) Tests of wrought-iron and of steel I-beams made at the Massachusetts Institute of Technology. Results of twenty-nine such tests are reported in Lanza's "Applied Mechanics," 1905 edition, p. 444 and p. 497.

(5) Tests of thirty-one steel I-beams and rolled girders made by Marburg at the University of Pennsylvania. These tests included I-beams of standard cross-section, I-beams with specially wide flanges, rolled by the Bethlehem Steel Company, and broad-flanged girder beams rolled by the same company. In Marburg's tests great care was taken to secure the greatest freedom of motion possible for the beam in the testing machine. The tests are reported in the Proceedings of the American Society for Testing Materials for 1909, p. 378.

(6) A test of a built-up plate girder made by Turneaure at the University of Wisconsin. This girder was so designed and tested that failure occurred by buckling of the web.

The results of the first five series of tests are summarized in Table 2. The last named test is reported in the Journal of the Western Society of Engineers for 1907, p. 788, and a summary of results is given in Table 8.

5. *I-beam Tests at the University of Illinois.*—The tests made in the Laboratory of Applied Mechanics of the University of Illinois and described in this bulletin include forty steel I-beams. These tests are summarized in Tables 3, 5 and 8. The general features of the tests may be indicated as follows:

(1) Ten 8-in., 18-lb. I-beams (Tests 1, 2, 7, 8, 12, 13, 18, 19, 32 and 33) were tested under loads applied at the one-third points of the span with spans varying from 5 ft. to 20 ft. These beams were so held in the testing machine as to afford the maximum possible freedom of motion for the beam (see Fig. 4).

(2) Eight 8-in., 18-lb. I-beams (Tests 3, 4, 9, 10, 14, 15, 20, and 21) were tested under conditions similar to those described in (1) except that the ends of the beams were firmly held so that they could not twist. (See Fig. 6.)

(3) Seven pairs of 8-in., 18-lb. I-beams (Tests 5, 6, 11, 16, 17, 22, and 23) were tested under conditions similar to those described in (1) except that the beams were firmly restrained against sidewise buckling. (See Fig. 7.)

(4) Four 8-in., 18-lb. I-beams (Tests 24, 25, 26, and 27) were tested with 10-ft. span. Two beams were loaded at the mid-point of the span, and two were loaded at the one-sixth points of the span. The beams were not restrained against sidewise buckling or against end twisting action. (See Fig. 4.)

(5) Two 8-in., 25.25-lb. I-beams (Tests 28 and 29) were tested with loads at the one-third points of a 10-ft. span. The beams were without sidewise or end restraint. (See Fig. 4.)

(6) Two 8-in., 18-lb. I-beams (Test 31) were tested under a load continued on the beams for 107 days. The computed stresses under the applied load were about equal to the yield-point strength of the material. The beams were symmetrically loaded at two points in a span of 8 ft. 9¾ inches.

(7) One pair of 8-in., 18-lb. I-beams (Test 30) with their webs fastened together by separators were tested with loads at the one-third points of a 10-ft. span.

(8) Six 12-in., 31.5-lb. I-beams (Tests 34-39) were tested over a span of 3 feet, with two symmetrical loads near mid-span. Different web conditions were obtained by varying the thickness of web by planing down the webs of some beams. These beams all failed in the web.

Nearly all tests were made on a four-screw 200,000-lb. Olsen testing machine with long table for beam tests. The instrument used for measuring deflections consisted of a framework supported entirely on the test beam, fitted with an extensometer dial by means of which deflection could be measured to one one-thousandth inch. For measuring longitudinal fiber deformation in the beams various forms of extensometers were used. A strain gauge of the Berry type proved the most satisfactory extensometer for this purpose.

The I-beams tested were bought in the open market at various times, and the beams may be expected not to differ far from the range of material found in practice.

6. *Yield Point of Structural Steel in Tension and in Compression.*—For I-beams the yield-point strength of the steel is, perhaps, the most important physical characteristic. The tension test is the most usual test for determining the physical properties of structural steel, and the yield point reported (frequently but incorrectly called the "elastic limit") is the yield point in tension. As flexural members of

structural steel may fail through the yielding of the compression fibers, the yield point of structural steel in compression seemed worthy of investigation. A considerable amount of test data are available on this subject, and the conclusion seems fairly well established that for the softer grades of steel the yield-point determined for tension is about the same as the yield-point in compression.* This conclusion is corroborated by the results of tests made in connection with the investigation of I-beams made at the University of Illinois. Table 1 gives the values of the stress at yield-point in tension and in compression for specimens cut from the flanges of I-beams and from flat bars. Both in the compression tests and in the tension tests the specimens were held in wedge-shaped grips, a special head being used in the compression tests, as shown in Fig. 1. The specimens were of such length that column action was not noticeable below the yield-point in the compression specimens.

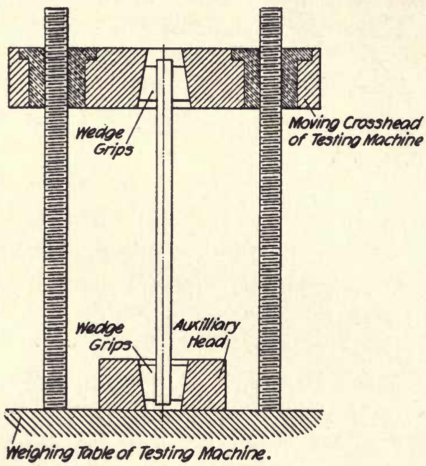

FIG. 1. APPARATUS FOR COMPRESSION TESTS OF STEEL.

The yield-point reported is the average of the values obtained by three methods: (1) the drop of the beam of the testing machine, which was well marked, as the speed used in testing below the yield-point was slow (0.1 in. per min.) and as the poise was easily kept in balance; (2) the "knee" of an autographic diagram of load and deformation, in which diagram the stretch or compression was magnified five times; and (3) the scaling of the specimens as shown by the flaking of the plaster of paris with which the specimen had been coated.

*J. B. Johnson, "The Materials of Construction," p. 502. This is a summary of tests by Chas. A. Marshall, and of tests at the Watertown Arsenal. Recent tests in British laboratories also show practically the same values for the yield-point strength of mild steel in tension and in compression.

7. *Failure of I-beams by Direct Flexure.*—In studying the failure of I-beams care must be exercised to distinguish between the primary failure and the final failure as judged by the shape of the beam after the test. Beams in which the primary cause of failure is excessive flexural stress not infrequently buckle sidewise *after* this excessive stress has weakened the flanges of the beam; in other cases the yielding of the flanges allows stress to be transferred to the web which then may twist or buckle. If a beam is held firmly against sidewise bending and has a thick web, the final failure under a load as applied in a testing machine will be by gradual sagging and the exact instant of failure will not be very clearly marked. Under excessive flexural fiber stress the time of application of load affects the deformation under load, and a beam may carry momentarily a load applied in a testing machine larger than that which if continued for several days would cause collapse of the beam.

In Table 2 are given test results obtained by various investigators in I-beam tests in which excessive flexural stress seems to have been the primary cause of failure. While it can not be certain that in every test the primary cause of failure was excessive flexural stress, yet, since every beam tabulated in Table 2 developed before failure computed stresses as high as might be expected for the yield-point strength of the material, and since in most cases friction of testing machine heads acted to prevent sidewise buckling, these tests appear to furnish a fairly satisfactory basis for the study of failure by direct flexure.

In Table 3 are given the test results for those I-beams tested at the University of Illinois for which excessive flexural fiber stress appeared to be the primary cause of failure. Figs. 10 to 15 inclusive (at the end of the text) give stress-deformation curves and stress-deflection curves for most of the beams tested at the University of Illinois.

A study of Table 2 and Table 3 shows that some beams which failed by direct flexure developed computed fiber stresses but little in excess of the usual values of yield-point strength of the material, while other beams developed momentarily stresses considerably higher. In Table 3 it will be seen that for nearly all the beams tested at the University of Illinois excessive deflection, large permanent set, or other sign of structural damage was observed at computed fiber stresses not much higher, if any, than the yield-point strength of the material. The load temporarily carried in a laboratory test depends in part on the speed of testing and the nature of the support of the beam in the testing machine. However, since a long-continued dead load or an oft-repeated live load would be more liable to injure a beam than an equal load ap-

plied by a testing machine, it is apparent that even under circumstances most favorable to the development of high fiber stresses, it is unsafe practice to regard as the ultimate fiber stress in flexure any value *higher* than the yield-point strength of the material of the beam.

Under long-continued static load the deformation of beams having their extreme fibers stressed to the yield-point of the material, increases for some time, frequently for several days, but the member does not necessarily fail. An illustration is furnished by a test made on two 8-in., 18-lb. steel I-beams loaded at two points each 12 inches to one side of the mid-point of an 8-ft. span. (Test No. 31, Fig. 15.) The load was applied to both beams until, as was shown by extensometers attached to the flanges, some fibers were stressed to the yield-point. Noticeable sidewise buckling of the beams had begun, and apparently failure was imminent. The load was kept constant for 107 days, and the extensometer on the most stressed flange was read from time to time. After a few days the fiber deformation reached a value practically constant, and the beam did not collapse during the test period. Thurston* reports tests showing similar results on transverse tests of steel of square section.

The excessive deflection of beams found when fibers are stressed beyond the yield-point, and the possibility of collapse emphasize the conclusion that for *I-beams the yield-point strength of the material in the flanges should be regarded as the ultimate fiber stress in flexure.* It is especially absurd to regard the ultimate tensile strength of steel as the ultimate fiber stress, as even under the most favorable conditions of service this fiber stress can not be developed before the I-beam collapses.

The results of those tests of I-beams in which failure occurred at stresses lower than the yield-point strength of the flange material due to sidewise buckling or to web failure, are tabulated and discussed in subsequent paragraphs.

8. *Inelastic Action of I-beams under Low Stress.*—Many recent writers on structural design have pointed out that in practically all steel structures the bending of beams and rods incidental to the erection of the structure and the use of drift pins in aligning rivet holes cause local stresses in excess of the yield-point strength of structural steel. These local stresses do not, however, cause the failure of the whole member. In Marburg's tests of I-beams and in those made at the University of Illinois frequent evidences of slight inelastic action at low stresses were observed. It is believed, however, that this inelastic action is the result of local stress and that it does not indicate the limit of load

*Thurston, "Text Book of the Materials of Construction," p. 516.

TABLE 1.

YIELD POINT OF STRUCTURAL STEEL IN TENSION AND IN COMPRESSION.

Specimens cut from flanges of I-beams and from flat bars. The length of specimens was about 9 in., 4 in. between grips, the width was from 1 in. to 1.5 in., and the thickness 0.25 in. to 0.50 in. The specimens from flanges of I-beams were planed down till the cross-section was rectangular.

Specimen from	Number of Specimens		Fiber Stress at Yield Point, lb. per sq. in.		Ratio of Yield Point in Tension to Yield Point in Compression
	Tension	Compression	Tension	Compression	
I-beam 16a	1	2	34,700	34,500	1.00
I-beam 16b	2	2	34,100	34,800	0.98
I-beam 26	2	2	32,500	34,600	0.94
I-beam 27	2	2	34,200	34,400	0.99
I-beam 24	2	2	34,700	35,000	0.99
I-beam 17a	2	2	36,300	35,100	1.03
I-beam 17b	2	2	35,400	36,100	0.98
I-beam 25	2	1	31,100	35,400	0.88
I-beam 21	2	2	33,800	32,600	1.03
I-beam 23	2	2	37,900	40,300	0.94
I-beam 14	2	2	36,400	32,000	1.14
I-beam 15	2	2	33,800	33,500	1.01
Flat 1	1	1	41,700	46,000	0.91
Flat 2	1	1	42,500	42,900	0.99
Flat 3	1	1	41,400	40,000	1.03
Flat 4	1	1	39,700	39,500	1.00
Flat 5	1	1	40,700	39,000	1.04
Flat 6	1	1	38,500	37,500	1.02
Flat 7	1	1	39,600	44,800	0.88
Flat X	1	1	37,900	35,000	1.08
	31	31			Av. 0.993

carrying capacity for the beam as a whole. In support of this belief the following facts are presented:

(1) In any test of material if apparatus of sufficient delicacy is used inelastic action can be detected at comparatively low stresses.*

(2) The physical properties of the material in various places in an I-beam may vary considerably;† the material at the root of the flange is usually weaker than the material in the flange or than the material in the web, and inelastic action of the beam under low stresses may be due in part to yielding of this weaker material while the flange material can develop further strength.

(3) If the load which at first application causes inelastic action be removed and reapplied, this second cycle of loading and removal of load will usually show much less inelastic action than is shown during the first cycle of loading and unloading, and successive cycles will show

*Moore, "The Physical Significance of the Elastic Limit," Proceedings of the Sixth Congress (1912) of the International Association for Testing Materials.

†Marburg, Proceedings of the American Society for Testing Materials, Vol. IX (1909), p. 385.

Moore, Proceedings of the American Society for Testing Materials, Vol. X (1910), p. 235.

Hancock, Proceedings of the American Society for Testing Materials, Vol. X (1910), p. 248; Vol. XI (1911), p. 477.

gradual improvement of the elastic qualities of the beam. Fig. 2 shows the computed fiber stresses and the deflections observed with successive applications of load in a test of an 8-in. I-beam. (Test No. 13, Table 5.) The energy lost in inelastic action for a cycle of loading and unloading is shown by the shaded area. It will be seen that during the third cycle the action of the beam was almost perfectly elastic. Only

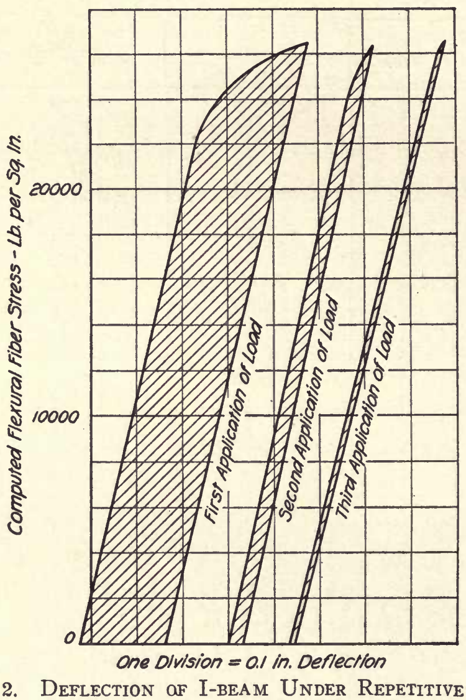

FIG. 2. DEFLECTION OF I-BEAM UNDER REPETITIVE LOADING.

a few moments elapsed between successive loadings so that rest of material could not have played an important part in the result. Similar results were obtained in tests of seven other beams.

For static load or for loads repeated at infrequent intervals and always acting in the same direction, local inelastic action does not seem important. The writer has not been able to discover any record of the failure by fatigue of metal originally sound in a bridge, building, or other structure designed to carry static load, but in structures under loads applied successively in opposite directions or loads rapidly repeated many millions of times it might be expected that local inelastic action may cause minute cracks or miscroscopic flaws which, spreading, would eventually cause the failure of the structure by fatigue of metal.

TABLE 2.

TESTS OF I-BEAMS—PRIMARY FAILURE BY DIRECT FLEXURE.

Test Made by	Number of Beams Tested	Depth, inches	Section	Material	Span, feet	Loading	Computed Fiber Stress at Failure, lb. per sq. in.	Modulus of Elasticity of Beam, lb. per sq. in.†
Burr and Elmore.	2	6	Medium Wt.	Wrought iron	4.00	Mid-point*	45,100	22,800,000
Burr and Elmore.	2	6	Medium Wt.	Wrought iron	5.00	Mid-point*	43,600	23,600,000
Burr and Elmore.	2	6	Medium Wt.	Wrought iron	6.00	Mid-point*	41,500	22,000,000
Tetmajer	1	3.98	Light Wt.	Wrought iron	2.62	Mid-point	62,800	28,800,000
Tetmajer	1	5.91	Light Wt.	Wrought iron	3.96	Mid-point	56,500	28,200,000
Tetmajer	3	7.87	Light Wt.	Wrought iron	5.25	Mid-point	53,800	28,200,000
Tetmajer	1	9.45	Light Wt.	Wrought iron	6.30	Mid-point	51,600	27,400,000
Tetmajer	1	11.81	Light Wt.	Wrought iron	7.87	Mid-point	53,200	26,500,000
Tetmajer	1	13.89	Light Wt.	Wrought iron	8.93	Mid-point	53,900	27,700,000
Tetmajer	1	15.75	Light Wt.	Wrought iron	10.50	Mid-point	52,500	27,700,000
Lanza	1	5	Light Wt.	Wrought iron	12.00	Mid-point	46,600	29,500,000
Lanza	1	6	Medium Wt.	Wrought iron	12.00	Mid-point	42,400	26,700,000
Lanza	1	6	Heavy Wt.	Wrought iron	14.58	Mid-point	37,800	27,900,000
Lanza	2	7	Heavy Wt.	Wrought iron	12.89	Mid-point	42,400	26,800,000
Lanza	4	7	Heavy Wt.	Wrought iron	14.20	Mid-point	41,200	28,800,000
Lanza	2	8	Medium Wt.	Wrought iron	13.58	Mid-point	45,900	28,000,000
Lanza	3	8	Heavy Wt	Wrought iron	14.58	Mid-point	38,300	27,500,000
Lanza	1	9	Light Wt.	Wrought iron	13.67	Mid-point	45,800	27,400,000
Lanza	1	9	Medium Wt.	Wrought iron	14.50	Mid-point	41,500	27,000,000
Lanza	2	9	Heavy Wt.	Wrought iron	14.67	Mid-point	38,900	27,100,000
Lanza	1	6	Medium Wt.	Steel	14.58	Mid-point	44,900	28,200,000
Lanza	1	7	Medium Wt.	Steel	12.91	Mid-point	42,100	27,500,000
Lanza	1	7	Medium Wt.	Steel	14.50	Mid-point	42,900	29,000,000
Lanza	2	8	Light Wt.	Steel	14.54	Mid-point	45,300	29,200,000
Lanza	1	9	Light Wt.	Steel	14.50	Mid-point	40,200	29,900,000
Lanza	4	10	Medium Wt.	Steel	14.00	Mid-point	40,200	28,200,000
Christie	1	8	Medium Wt.	Steel	4.92	Mid-point	45,200	30,900,000
Christie	1	8	Medium Wt.	Steel	3.25	Mid-point	45,100	25,000,000
Christie	2	8	Heavy Wt.	Steel	20.00	Mid-point	44,200	29,000,000
Christie	1	8	Heavy Wt.	Steel	12.00	Mid-point	41,300	27,500,000
Marburg	3	15	Lt. Wt. Beth'm I-beam	Steel	15.00	Mid-point	46,100	26,900,000
Marburg	3	15	Lt. Wt. Stand. I-beam	Steel	15.00	Mid-point	42,200	26,200,000
Marburg	3	15	Lt. Wt. Beth'm Girder	Steel	15.00	Mid-point	53,900	26,900,000
Marburg	3	15	Lt. Wt. Beth'm I-beam	Steel	15.00	One-quarter pts.*	37,900	26,400,000
Marburg	3	15	Lt. Wt. Beth'm Girder	Steel	15.00	One-quarter pts.*	41,100	27,200,000
Marburg	3	24	Lt. Wt. Stand. I-beam	Steel	20.00	One-quarter pts.*	33,000‡	25,800,000
Marburg	3	24	Lt. Wt. Beth'm Girder	Steel	20.00	One-quarter pts.*	34,300‡	25,600,000
Marburg	1	30	Lt. Wt. Beth'm I-beam	Steel	20.00	One-quarter pts.*	32,300‡	29,400,000

*Beams mounted in testing machine so as to give freedom for sidewise motion.
†Computed from deflection readings.
‡Fiber stress in flexure at failure greater than the yield-point strength of test pieces cut from beam flanges.

TABLE 3.

TESTS OF I-BEAMS AT THE UNIVERSITY OF

In all tests included in this table the beams developed computed fiber stresses equal to

Test No.*	Beam	Span, feet	Loading	Yield-point Strength of Material in Flanges,° lb. per sq. in.
5	Pair of 8-in., 18-lb. I-beams........ Restrained from sidewise buckling.	5	⅓ points	35,300
6	Pair of 8-in., 18-lb. I-beams........ Restrained from sidewise buckling.	5	⅓ points	35,300
4	8-in., 18-lb. I-beam................. Restrained from end twisting.	5	⅓ points	33,800
1	8-in., 18-lb. I-beam................ No restraint.	5	⅓ points	37,000
2	8-in., 18-lb. I-beam................. No restraint.	5	⅓ points	37,000
11	Pair of 8-in., 18-lb. I-beams........ Restrained from sidewise buckling.	7.50	⅓ points	35,200
10	8-in., 18-lb. I-beam................. Restrained from end twisting.	7.92	⅓ points	33,900
16	Pair of 8-in., 18-lb. I-beams........ Restrained from sidewise buckling.	10	⅓ points	34,300
17	Pair of 8-in., 18-lb. I-beams......... Restrained from sidewise buckling.	10	⅓ points	35,900
15	8-in., 18-lb. I-beam................ Restrained from end twisting.	10	⅓ points	33,800
12	8-in., 18-lb. I-beam................. No restraint	10	⅓ points	33,800
30	Pair of 8-in., 18-lb. I-beams......... With separators.	10	⅓ points	32,400
28	8-in., 25.25-lb. I-beam.............. No restraint.	10	⅓ points	34,100
26	8-in., 18-lb. I-beam................. No restraint.	10	Mid-point	32,500
27	8-in., 18-lb. I-beam................. No restraint.	10	Mid-point	34,200
22	Pair of 8-in., 18-lb. I-beams........ Restrained from sidewise buckling.	15	⅓ points	34,200
23	Pair of 8-in., 18-lb. I-beams........ Restrained from sidewise buckling.	20	⅓ points	35,300

*See Fig. 10-15 at the end of the bulletin.

°Determined from tests of specimens cut from flanges.

TABLE 3 (Continued)

Illinois—Primary Failure by Direct Flexure.

or greater than the yield-point strength of the material.

Computed Fiber Stress at Maximum Applied Load, lb. per sq. in.	Deflection under Maximum Load, inches	Modulus of Elasticity, lb. per sq. in.°°	Remarks
41,500	0.30	26,500,000	0.28 in. set† after fiber stress of 33,400 lb. per. sq. in. Beams sagged gradually.
41,500	0.33	28,000,000	0.18 in. set after fiber stress of 33,400 lb. per sq. in. Beam sagged gradually.
35,300	....	....	Final failure by sidewise buckling *after* yield-point strength of material was developed.
38,400	0.20	....	Final failure by sidewise buckling *after* yield-point strength of material was developed.
37,800	0.13	30,300,000	Final failure by sidewise buckling *after* yield-point strength of material was developed.
37,300	....	....	Beam sagged gradually, excessive fiber deformation at 34,300 lb. per sq. in. fiber stress.
34,300	....	....	Final failure by very gradual sidewise buckling *after* yield-point strength of material was developed.
35,200	0.76	27,800,000	0.12 in. set after fiber stress of 33,700 lb. per sq. in. Beams sagged gradually.
36,600	0.70	26,300,000	0.08 in. set after fiber stress of 31,000 lb. per sq. in. Beams sagged gradually.
35,400	0.76	25,800,000	Ultimate strength apparently reached. Beam sagged gradually.
36,000	0.58	30,000,000	Final failure by sidewise buckling *after* yield-point strength of material was developed.
38,200	0.55	30,400,000	0.11 in. set after fiber stress of 34,800 lb. per sq. in. Final failure by sidewise buckling *after* yield-point strength of material was developed.
39,800	0.78	27,800,000	Flanges showed signs of crippling at fiber stress of 33,400 lb. per. sq. in.
36,200	0.63	28,400,000	Final failure by sidewise buckling *after* yield-point strength of material was developed.
37,600	0.45	28,000,000	Final failure by sidewise buckling *after* yield-point strength of material was developed.
35,300	....	....	Restraining batten plates clamped to beams. Hold of clamps loosened and final failure of beams was by sidewise buckling after yield-point strength of material was developed.
36,500	2.10	24,000,000	0.54 in. set after fiber stress of 31,600 lb. per sq. in. Beams sagged gradually.

°°Determined from deflection of beams.

†Set denotes deflection remaining after the removal of load from the beam.

9. *Buckling of Compression Flanges of I-beams: Equivalent Column Length.*—In an I-beam under a load which acts in the plane of the web there is a tendency for the compression flange to buckle sidewise due to column action. On account of this column action the ultimate available fiber stress for a beam with no restraint against sidewise buckling, as calculated by the flexure formula, may be less than the yield-point strength of the material in the flanges. Column action in a beam is more complex than the action of a strut under direct

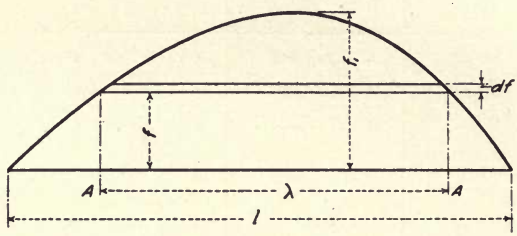

FIG. 3. DIAGRAM OF STRESS IN COMPRESSION FLANGE.

compression. In a strut a compressive load P is applied at one end and the average stress over the cross section due to the direct compression is the same all along the strut. For a strut the average stress over any cross section is given by the equation $f_c = \frac{P}{A}$, where P is the total compressive load, A the cross sectional area of the strut, and f_c the average stress due to direct compression. In a beam the compressive stress in the extreme fibers of the compression flange due to flexural action varies according to the location of the section, increasing from zero at an end support (or point of inflection for a fixed-ended beam) to a maximum at some point in the span. The compression flange of the I-beam then may be regarded as a strut loaded with a large number of elementary loads dP applied at successive points along the span. Each elementary load or increment of load may be considered to produce an increment in the compressive stress of the flange. The value of the compressive stress at any section will then be the summation of the increments of compressive stress from the point of zero stress, or $f = \int df$. This stress f for the remotest fiber of any section will be very nearly equal to the flexural stress produced by the flexure of the beam, which stress is computed by the usual flexure formula $f = \frac{Mc}{I}$, in which M = the bending moment at the point of span considered, I = the moment of inertia of cross-section of beam, and c = distance from the neutral surface of the beam to the extreme compression fiber. To

find the effect of this kind of column loading it will be necessary to deduce an expression for the value of the compressive stress due to column action which will be developed with a column loaded with increments of load as here considered.

For a strut under direct compression the average compressive stress developed at failure f_c may be expressed fairly well by the formula

$$f_c = f_e - k\frac{l}{r}* \dots\dots\dots\dots\dots\dots (1)$$

in which

f_e = a value of stress about equal to the yield-point strength of the material,

l = length of strut,

r = minimum radius of gyration of cross-section of strut,

k = an experimentally determined constant.

We may regard the term $k\frac{l}{r}$ in either of two ways, (1) as representing a reduction from f_e of available ultimate fiber stress in the strut, due to the effect of column action, or (2) as a fiber stress which is due to a bending action in the column, produced by the same load as produces f_c, and which added to f_c brings the extreme fiber stress up to f_e. By the second conception the stress f_e is the sum of the direct compressive stress f_c and the stress due to column bending.

For the purposes of this discussion it will be convenient to use the second conception; i. e., to consider the last term as a stress produced by a bending action in the column.† In the case of the I-beam, the stress in the remotest fiber of the compression flange on the edge having the highest compression may be considered to be made up of the sum of the flexural fiber stress f_1 (computed by the usual flexure formula, $f_1 = \frac{Mc}{I}$) and the column bending stress f'. This stress will be a maximum at the section where the bending moment will be a maximum, and at failure by side buckling of the flange we may consider that $f_e = f_1 + f'$, where f_1 is the computed flexural fiber stress at the dangerous section, and f_e has a value not greatly different from the yield point of the material.

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*While this "straight line" formula for columns is based directly on experiment rather than on mathematical reasoning, it is generally accepted as expressing with a good degree of accuracy the law of failure for columns whose $\frac{l}{r}$ is not greater than about 150 and which are of sufficiently stocky construction to avoid danger of failure by "wrinkling" of parts or local collapse.

†Other methods of analytical treatment of the sidewise buckling of I-beams have been proposed. Some of them are based on the Rankine-Gordon-Schwartz column formula, others on reasoning analagous to that used in developing the Euler column formula.

See Michell, in the Philosophical Magazine for 1899, p. 298; Reissner, in the American Machinist for March 10, 1906; H. D. Hess, in the Proceedings of the Engineers' Club of Philadelphia for April, 1909; Boyd, "Strength of Materials," p. 219; Cambria and Carnegie Steel Handbooks.

To determine the column bending stress f', it will be necessary to take into account the manner of application or of distribution of the assumed compressive loading along the length of the compression flange. At any point along the compression flange the column load may be taken to be the flexural fiber stress at that point, since by this conception the amount of the column load per unit of area of section is the flexural fiber stress. This flexural fiber stress increases from the point of zero bending moment in the beam to the dangerous section. To determine the effect of column action, the increment or differential of flexural fiber stress df applied along a differential of length of flange $d\lambda$ (which is the only column load which acts throughout the column length λ) will be considered as producing column bending stress at the dangerous section. This increment of load is applied at any two points A and A (Fig. 3) and acts upon a column of the length λ (the calculated flexural fiber stress being the same at A and A). The sum of all the loads on the whole length of column ($\int df$) will be the flexural fiber stress at the dangerous section (f_1). To determine the total column bending stress f' at the dangerous section A it will be necessary to make a summation of the effects of the increments of column load df over the length of flange to be considered. By analogy with the straight-line column formula, adopting q as the coefficient for the formula as applied to sidewise buckling of the flange, the term expressing the column bending stress will be of the form $q\frac{\lambda}{r'}$, where r' is the radius of gyration of a cross-section of the compression flange about a gravity axis parallel to the depth of the web. As the elements of column load df vary along the flange and as the proportional effect of each elementary load as compared with the sum of all the loads must be used in the summation, it is necessary to introduce the ratio $\frac{df}{f_1}$ into the term. Then

$$f' = \int q\frac{\lambda}{r'}\frac{df}{f_1} = \frac{q}{r'}\int \lambda\frac{df}{f_1} \quad \ldots\ldots\ldots\ldots (2)$$

This is the column bending stress at the dangerous section.

It will be found convenient to consider this stress as equal to the column bending stress in an ordinary strut loaded with a load f_1, and having a length of ml, where m is a coefficient depending upon the method of loading and conditions of continuity and l is the length of the beam. ml may be called the equivalent column length. Equation (2) may then be written

$$f' = \frac{q}{r'}\int \frac{\lambda\, df}{f_1} = q\frac{ml}{r'} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The equation for the computed flexural fiber stress at failure due primarily to sidewise buckling will be

$$f_1 = f_e - q\frac{ml}{r'} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

where f_e = a value not greatly different from the yield-point strength of the material in the flange.

q = a coefficient of column action.

ml = the equivalent column length of the flange of the beam, the coefficient m being found by equation (3) for different loadings and different conditions of continuity.

r' = the radius of gyration of the compression flange about a gravity axis parallel to the web. For practical purposes r' may be taken as the radius of gyration of the I-section about a gravity axis parallel to the web.

From equation (3) an expression for ml may be written $ml = \int \frac{\lambda df}{f_1}$. It may help in integration to consider that $\int \lambda df$ is the same as the area under the curve of flexural stress for the full length of the beam in simple supported beams, as is indicated in Fig. 3. Then, if f_a is the *mean* ordinate of the curve of flexural stress, $\int \lambda df = f_a l$.

$$ml = \int \frac{\lambda df}{f_1} = \frac{f_a l}{f_1 l}$$

That is, the equivalent column length of the compression flange of an I-beam is equal to the span multiplied by the ratio of the mean flexural fiber stress in the compression flange to the flexural fiber stress in the compression flange at the dangerous section. For a uniformly loaded beam of constant cross section m is found to be $\frac{2}{3}$. For a beam with a single load at any point between supports m is $\frac{1}{2}$. Table 4 gives values of m for various beam loadings. For beams fixed at the ends it will be

TABLE 4.

SIDEWISE BUCKLING OF I-BEAMS:

VALUES OF THE COEFFICIENT m FOR VARIOUS LOADINGS OF BEAMS.

Loading	Value of m
Simple beam, uniform load	0.667
Simple beam, mid-point load	0.500
Simple beam, single concentrated load at any point of span	0.500
Simple beam, one-third point loads	0.667
Simple beam, one-quarter point loads	0.750
Simple beam, one-sixth point loads	0.888
Cantilever beam, uniform load	0.667
Cantilever beam, end load	1.000
Fixed-ended beam, uniform load	0.281
Fixed-ended beam, mid-point load	0.250

seen that one limit for λ will be the distance between points of inflexion of the elastic curve.

10. *Buckling of Compression Flanges of I-beams; Tests.*—The value of the coefficient q of equation (4) is to be determined from the results of flexure tests of I-beams. All available data of tests of I-beams were studied and the test results given in Table 5 were chosen as furnishing the best basis for the determination of q. In selecting data suitable for the study of resistance to sidewise buckling, only those tests were considered in which the primary cause of failure was evidently sidewise buckling. Beams which were wholly or partially restrained laterally by the method used in supporting them in the testing machine were not considered. Fig. 4 shows the method used at the University of Illinois for supporting beams and preserving freedom in respect to sidewise buckling. Beams in which the yield-point strength of the material in the flanges was developed before failure were not considered, as the mere presence of such a fiber stress would explain the failure of a beam and might readily be the cause of sidewise buckling, which would then be a secondary and not a primary failure. The non-consideration of beams free to buckle sidewise which develop fiber stresses as great as the yield-point strength of the material affected only beams of short span or beams of medium span in which the flange material was unusually weak. As a matter of fact the results obtained for resistance to sidewise buckling would not be materially affected whether such beams were considered or not. In making up Table 5 tests were not considered in which web failure seemed to be the primary failure.

Fig. 5 shows graphically the results of the tests given in Table 5. The computed fiber stress at failure (generally called the modulus of rupture) was chosen as a criterion of the strength of an I-beam, rather than the "elastic limit" of the beam, for the following reasons: (1) The failure of beams which buckle sidewise is sharply marked, and the personal equation of the observer will affect the determination of the point of failure but slightly. On the other hand any determination of the elastic limit is dependent upon the sensitiveness of apparatus used in obtaining readings of deformation and upon the interpretation of a plotted curve, and it is much more subject to variations due to personal equation than is the computed fiber stress at failure. (2) The load at failure is more dependent upon the average physical properties of the beam material and less on local stresses and individual peculiarities than is the elastic limit. As the yield point of the material was not exceeded, the computed fiber stress at failure may be considered to vary but little from the actual fiber stress.

TABLE 5.

Tests of I-Beams—Primary Failure by Sidewise Buckling.

In all the tests of beams recorded in this table primary failure occurred by sidewise buckling of the compression flange at computed flexural stresses less than the yield-point strength of the material in the flanges.

Beam	Tested by	Test Numbers*	Number Tested	Span ft.	Loading	Slenderness Ratio for Whole Span l/r'	Equivalent Slenderness Ratio for Sidewise Buckling $\frac{ml}{r'}$	Computed Fiber Stress at Failure lb. per sq.in.	Modulus of Elasticity † lb. per sq.in.
15-in., 42-lb. I-beam	Marburg	......	8	15.00	One-quarter points	166	125	34,700	26,900,000
24-in., 72-lb. Beth'm I-beam	Marburg	......	8	20.00	One-quarter points	129	97	34,600	26,400,000
8-in., 18-lb. I-beam	Univ. of Ill.	7, 8	2	7.50	One-third points	107	71	36,600	25,100,000
8-in., 18-lb. I-beam	Univ. of Ill.	13, 32, 33	3	10.00	One-third points	143	95	32,000	28,600,000
8-in., 18-lb. I-beam	Univ. of Ill.	18	1	15.00	One-third points	214	143	28,900	....
8-in., 18-lb. I-beam	Univ. of Ill.	19	1	20.00	One-third points	286	191	28,100	32,300,000
8-in., 18-lb. I-beam	Univ. of Ill.	3	1	5.00	One-third points restrained against end twisting	71	47	31,400‡	27,600,000
8-in., 18-lb. I-beam	Univ. of Ill.	9	1	7.92	One-third points restrained against end twisting	113	75	34,300	....
8-in., 18-lb. I-beam	Univ. of Ill.	14	1	10.00	One-third points restrained against end twisting	143	95	33,800	30,200,000
8-in., 18-lb. I-beam	Univ. of Ill.	20	1	15.70	One-third points restrained against end twisting	224	150	32,800	....
8-in., 18-lb. I-beam	Univ. of Ill.	21	1	20.00	One-third points restrained against end twisting	286	191	29,200	25,000,000
8-in., 18-lb. I-beam	Univ. of Ill.	24, 25	2	10.00	One-sixth points	143	119	31,500	31,400,000
8-in., 25.25-lb. I-beam	Univ. of Ill.	29	1	10.00	One-third points	150	100	36,500	27,100,000

*See Figs. 10-15 at the end of bulletin.

†Computed from deflections of beams.

‡Very low stress at failure probably due to additional stress caused by imperfect bearing of restraining devices.

The advisability of adjusting the fiber stresses developed in tests of I-beams to compensate for variation in strength of material in the flanges of different test beams was considered, but it was decided to base conclusions on the stresses computed for the tests. Two reasons led to this decision: (1) Due to cold-straightening and other bending which a beam receives there is considerable variation in strength in different parts of the same beam, and the strength of test pieces from one part of the beam would not be wholly representative of the strength of other parts. (2) For beams of long span the resistance to sidewise buckling is dependent not so much on the strength of material as on its stiffness (of which the modulus of elasticity is an index); for beams of medium span the resistance to sidewise buckling is dependent partly on the strength of material and partly on its stiffness; hence the proper adjustment of stresses to compensate for variation of material would be a matter of no small difficulty.

From Fig. 5 it may be seen that the equation

$$f_1 = 40{,}000 - 60\,\frac{ml}{r'} \quad \ldots\ldots\ldots\ldots\ldots (5)$$

represents the results with a fair degree of accuracy. The extreme values observed fall within 2,500 lb. per sq. in. of the values given by the above equation.

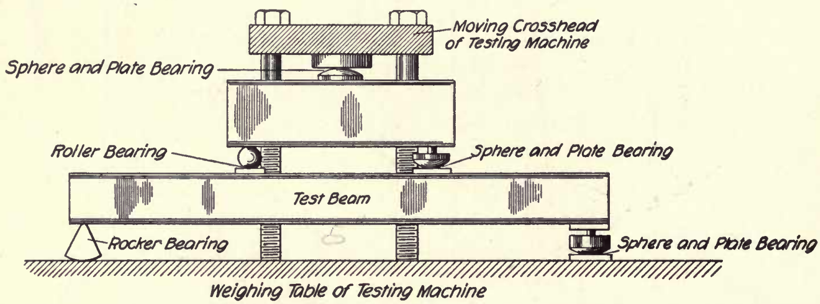

FIG. 4. APPARATUS FOR TESTING I-BEAM WITHOUT RESTRAINT OF ENDS OR OF COMPRESSION FLANGE.

A comparison of equation (4) with the results of tests of columns is of interest. Tests made by J. E. Howard at the Watertown Arsenal* on H-section steel columns with pin ends have been chosen as tests which furnish an excellent basis of comparison of column test results and I-beam test results. The results of Howard's tests of H-section columns with pin ends may be expressed by the equation

$$P/A = 36{,}000 - 100\,\frac{l}{r} \quad \ldots\ldots\ldots\ldots (6)$$

*Tests of Metals for 1909, p. 754; Proceedings of the American Society for Testing Materials, Vol. IX (1909), p. 413.

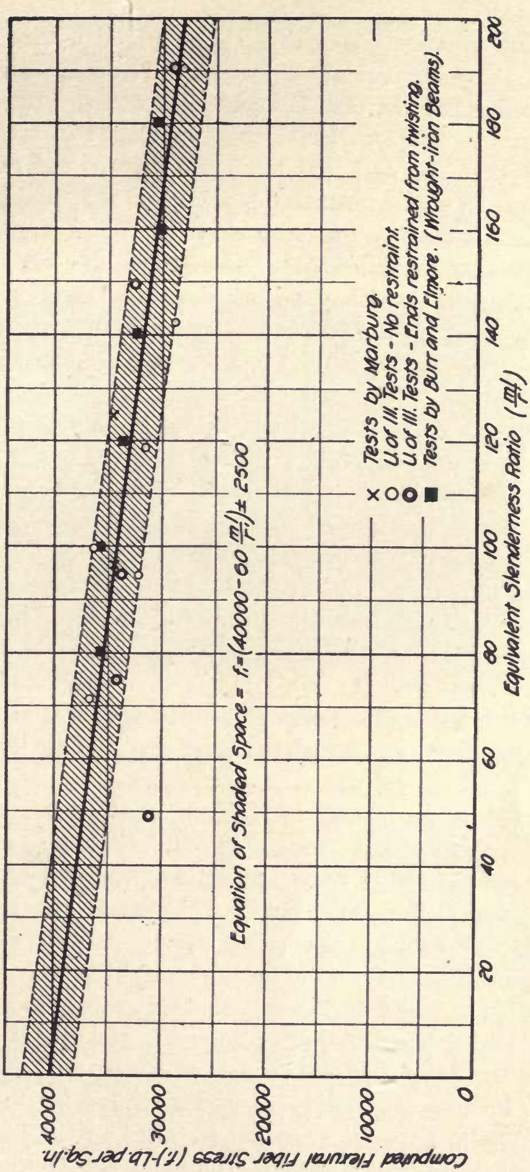

Fig. 5. Results of Tests for Sidewise Buckling of I-beams.

in which P/A is the average intensity of compressive stress at failure, l is the length of the column and r is the least radius of gyration of the column section.

Comparing equation (5) with equation (6) it is seen that, as might be expected, the coefficient of equivalent slenderness ratio for the beam formula is somewhat less than the coefficient of slenderness ratio for the column formula. The smaller value of coefficient in the beam formula is doubtless due to the fact that in a beam there is end restraint against sidewise buckling and a restraining action of the web and the tension flange. The first term in the beam equation (5) is larger than the first term in the column equation (6). The flanges and webs of the I-beams were rolled thinner than the flanges and webs of the H-sections, and the additional work of rolling done on the I-sections may explain the increase in the yield-point strength of the material over that of the H-sections.

It should be noted that all but one of the beams given in Table 5 are "light" sections. The web and the tension flange of "heavy" I-beams would offer more restraint against sidewise buckling than do the web and the tension flange of "light" I-beams, and the fiber stresses developed at failure may reasonably be expected to be higher. Such a result is indicated by the tests made by Burr and Elmore at Rensselaer Polytechnic Institute, to which reference has already been made. The results of these tests are shown in Fig. 5 by small black squares. The tests were made on medium-weight wrought-iron I-beams, 6 in. deep, and it is seen that the greater strength and stiffness of steel I-beams was about offset by the greater stockiness of section of the Burr and Elmore wrought-iron test beams.

As test data are lacking for "heavy" steel I-beams, as the formula (equation 5) derived from tests of "light" I-beams gives results which err on the side of safety when applied to "heavy" I-beams, and as the reduction below yield-point strength of material of fiber stress at failure is not large for ordinary spans, no attempt will be made to derive a separate formula for I-Beams of medium-weight or heavy-weight sections.

Attention is called to the fact that in no case should the ultimate flexural stress be taken as higher than the yield-point strength of the material in the flanges. In the absence of special tests of material 35,000 lb. per sq. in. may be used as an average value for the yield point of structural steel. Especial attention is called to the fact that equation (5) gives ultimate values of fiber stress and not working values, which should, of course, be much lower.

11. *Tests to Failure of Beams Restrained from Twisting of Ends and Beams Restrained from Sidewise Buckling.*—Two series of tests were

carried out for the purpose of investigating the action of I-beams restrained against end twisting and of beams restrained against sidewise buckling. Fig. 6 shows the arrangement of apparatus used in testing beams restrained against end twisting. To each end of the web of the test beam heavy angles were bolted by one leg and the other leg of each angle was bolted to an end piece which rested on a roller. When the test beam was placed in the testing machine evenness of bearing under rollers was secured by the use of thin metal shims. This method of supporting the test beams proved effective in preventing end twisting and did not affect the tendency of the beam to buckle sidewise.

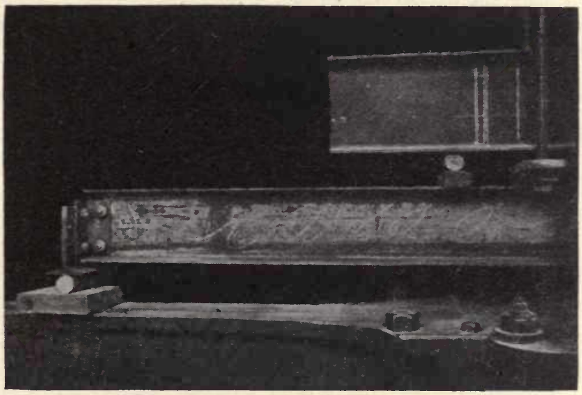

FIG. 6. APPARATUS FOR TESTING I-BEAM WITH RESTRAINT AGAINST END TWISTING.

The fiber stresses developed at failure in those beams which were restrained against end twisting are given in Tables 3 and 5. Short-span beams so restrained did not develop quite so high stresses at failure as did similar beams tested without restraint against end twisting. This was probably due to imperfect bearings at the ends. For all except the short-span I-beams the fiber stresses developed at failure by beams restrained from end twisting did not differ appreciably from the fiber stresses developed at failure by the beams not so restrained. Failure for both kinds of beams occurred by sidewise buckling, and it would seem that restraint of ends of I-beams against twisting does not appreciably increase their resistance to sidewise buckling.

The method of testing beams restrained against sidewise buckling is shown in Fig. 7. For each test two beams were fastened together along their compression flanges by means of batten plates spaced about ten inches apart. Each batten plate was fastened to the flanges of the beams by four studs of cold-rolled steel fitting snugly in drilled holes. This

device prevented appreciable sidewise buckling and all beams thus restrained failed very gradually by vertical sagging with the exception of the beams with 15-ft. span in which the batten plates were merely clamped to the flanges of the beams, and in which, though the full yield-point strength of the material was developed, the beams finally buckled sidewise. In all beams tested with restraint against sidewise buckling the maximum computed fiber stress developed in the test was equal to or slightly greater than the yield-point strength of the material in the flanges. It would seem that for beams effectively restrained against sidewise buckling the fiber stress developed before failure in flexure will be as great as the yield-point strength of the material, regardless of the length of the span. What constitutes effective restraint is discussed in the next paragraph.

TABLE 6.

EFFECT ON THE ELASTIC LIMIT OF I-BEAMS OF RESTRAINT AGAINST TWISTING OF ENDS AND AGAINST SIDEWISE BUCKLING.

All tests made on 8-in., 18-lb. I-beams loaded at the one-third points of the span.

Span ft.	Number of Beams Tested for Each Item	Computed Fiber Stress at the First Observed Elastic Limit lb. per sq. in.		
		No Restraint	Restrained against Twisting of Ends	Restrained against Sidewise Buckling
5	2	27,300	23,000	22,300
7.5	2	27,900	23,300	26,300
10	2	23,000	19,300	26,600
15	1	25,200	22,000	21,200
20	1	21,000	23,400	24,000

12. *Effectiveness of Sidewise Restraint of I-beams.*—In the tests of 8-in. I-beams, measurements of the extreme fiber deformation in the flange (stretch and shortening) at mid-span were made. By plotting the observed fiber deformations against the fiber stress computed by the usual flexure formulas, curves were obtained showing local action of the beams under load. These curves (Fig. 10-15) are given at the end of the bulletin. From these curves fiber stress at the elastic limit first observed at any part of the beam was determined,* and these stresses have been tabulated in Table 6. An examination of this table shows that inelastic action was detected in some restrained beams at computed fiber stresses lower than was the case for the corresponding unrestrained beams, and that, in general, the effect of restraint on elastic limit is not great. A reasonable explanation of this would seem to be that the re-

*The elastic limit was located by the method proposed by the late Prof. J. B. Johnson. His method consists in finding the point on a stress-deformation curve at which the deformation is increasing fifty per cent more rapidly than its initial rate of increase.
See Johnson, "The Materials of Construction," pp. 18-20.

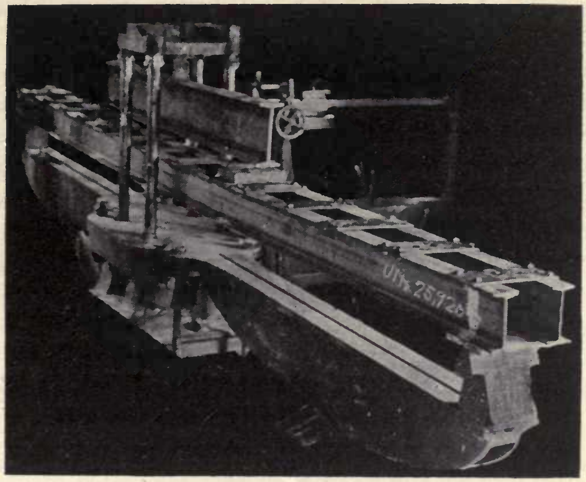

FIG. 7. APPARATUS FOR TESTING I-BEAM WITH RESTRAINT AGAINST SIDEWISE BUCKLING.

straining devices sometimes introduce additional stresses into the beam. As noted in "11. Tests to Failure of Beams Restrained from Sidewise Buckling and Beams Restrained Against Twisting of Ends," restraint against twisting of ends produced no marked effect on the *ultimate* strength of medium-span and long-span I-beams, while with restraint against sidewise buckling the fiber stresses developed before failure, even for the longest-span beams, were as high as the yield-point strength of the material.

The elastic limit observations in connection with the observations of ultimate fiber stress seem to indicate that the resisting effect exerted by restraint is not noticeable until failure is imminent. Observations on sidewise deflection tend to confirm this conclusion. In the tests of certain of the unrestrained I-beams the sidewise deflection was measured. Table 7 records the sidewise deflection observed at loads which give a computed flexural fiber stress of 16,000 lb. per sq. in. (an ordinary working stress). It will be seen that the sidewise deflection is small, and generally in the tests it continued to be small until failure of the beam became imminent. So small is this sidewise deflection for working stresses that it is questionable whether such restraining members as usually would be attached to beams in actual structures will be stiff enough to prevent it. In structures the usefulness of restraining I-beams against sidewise buckling lies mainly in the fact that such restraint renders available the full yield-point strength of the material in the flanges of the beams, and that should failure occur, with effective sidewise and

end restraint, the beam will fail by gradual sagging rather than by sudden collapse.

Any restraining devices to prevent sidewise buckling of I-beams should provide resistance to sidewise bending moments. Separators between the ribs of a pair of beams do not provide such resistance. A pair of beams held together merely by bolts and separators was tested (Test No. 30), and though the yield-point strength of the material was developed, final failure occurred rather suddenly by sidewise buckling. While comparatively slight sidewise restraint may enable a beam to develop the full yield-point strength of the material, a strong, stiff restraining system is needed to prevent sudden collapse when final failure does occur. Another illustration of this was furnished by testing a pair of 8-in., 18-lb. I-beams having a span of 15 ft. (Test No. 22) and restrained against sidewise buckling. In this test the batten plates holding the beams together were merely clamped to the flanges and not bolted. In the test the full yield-point strength of the flange material was developed, but soon afterward the beams failed quite suddenly by sidewise buckling. The clamps were not strong enough to hold the batten plates to the beam flanges under the large sidewise force developed when the yield point of the material in the beam was reached.

13. *Web Failure of I-beams.*—I-beams and built-up girders of short span are sometimes in danger of failure by crippling of the web. Web failure may be caused in several ways: (1) The fiber stress in shear at the middle of the web may exceed the yield-point strength in shear of the web material. (2) Accompanying the shearing fiber stress at any point of the web is a compressive stress of equal intensity acting in a direction inclined at 45 degrees with the direction of the shearing stress, and this compressive stress may become so great as to cause buckling. (3) There may be an excessive compressive stress near the junction of web and flange and adjacent to a concentrated load or reaction. The shapes assumed by a cross-section of an I-beam after web failure are shown in Fig. 8. The shape and position at (a) is that due to torsion of the beam as a whole; that at (b) to buckling of the web; and that at (c) to local compressive stress at root of flange. What has been referred to previously as failure by twisting of ends of I-beams is in most cases primarily caused by excessive local compression at the root of the flange.

An approximate method of computing the compressive stress at the root of the flange adjacent to a concentrated load or an end reaction, has been given by C. W. Hudson* as follows: Imagine a small piece cut

*Engineering News, December 9, 1909.

TABLE 7.

SIDEWISE DEFLECTION OF I-BEAMS AT A COMPUTED FIBER STRESS OF 16,000 LB. PER SQ. IN.—BEAMS FREE TO MOVE LATERALLY.

Beam	Material	Span ft.	Loading	Deflection in.
8-in., 18-lb. I-beam......	Steel.........	10	One-third points.....	0.046
8-in., 18-lb. I-beam......	Steel.........	20	One-third points.....	0.036
8-in., 25.25-lb. I-beam...	Steel.........	10	One-third points......	0.026
8-in., 25.25-lb. I-beam...	Steel.........	10	One-third points.....	0.019
17.5-in. built-up beam...	Wrought iron	12.9	One-third points.....	0.080
24-in. built-up beam.....	Wrought iron	14.2	Load 12 in. each side of center.........	0.067

from the flange and web of an I-beam immediately over a bearing block (as shown in Fig. 9), and imagine this piece to be held in equilibrium by the elastic forces which act on it while it is in its place in the beam. The forces are (1) the pressure of the reaction at the bearing block P; (2) the compression in the web which equals $f_w tb$, when $f_w =$ the average intensity of compressive stress, $t =$ the thickness of web, and $b =$ the length of bearing block; (3) a horizontal shearing force S_h; and (4) a vertical shearing force S_v. Very little of the total shear would be balanced by the small internal shearing stress in the flange of an I-beam, and if the section considered be taken at the root of the flange we may write without serious error

$$S_v = S_h = 0$$

Then the compressive stress on the web is balanced by the reaction on the bearing block. The compressive stress may be regarded as uniformly distributed, and we may write

$$f_w = \frac{P}{bt} \qquad \ldots\ldots\ldots\ldots (7)$$

In the above discussion the case considered is for the compressive stress adjacent to an end reaction. The reasoning for the compressive stress in the web adjacent to a concentrated load would be similar.

The compressive stress in the web of an I-beam necessary to cause buckling of the web is computed in most text books on strength of materials on the assumption that the web of the I-beam is in the same condition of stress as a fixed-ended column whose length is equal to the vertical distance between flanges multiplied by the secant of 45 degrees, and whose radius of gyration is equal to the thickness of the web divided by $\sqrt{12}$, and in which the average intensity of compressive stress is equal to the maximum intensity of shearing stress in the web of the I-beam. This shearing stress is very nearly equal to the total shear divided by the area of the web. The assumption of fixed-ended conditions and the

neglect of the restraint against the buckling of the web by tensile stress in the lower part of the beam render the accuracy of this method somewhat uncertain.

14. *Web Failure of I-beams; Tests.*—After gathering the data of tests it is realized that the whole amount of data on web failure of I-beams is small. The drawing of conclusions from these data is further complicated by the fact that several web failures of test beams seemed to be due partly to shearing stress in the web and partly to compressive stress in the web adjacent to bearing blocks.

In selecting test data for the study of web failure of I-beams, only those tests were taken in which at failure the fiber stress in the flanges was less than the yield-point strength of the material and in which the failure took place by crippling of the web.

Six of the tests selected were made at the University of Illinois. As noted on p. 33, variation in web dimension was obtained by planing down the webs of some of the beams.

The results of the tests selected for the study of web failure are given in Table 8. In the tenth line of the table is given the slenderness ratio of the web computed on the assumptions usually made in text books on mechanics of materials and named in the preceding paragraph. In the thirteenth line of the table is given the computed fiber stress at failure of the web by buckling as determined by Euler's formula for fixed-ended columns. Euler's formula was chosen on account of the high values of slenderness ratio. It will be seen that the calculated compressive stresses corresponding to the loads carried (tabulated in the twelfth line of the table) were in three cases very much in excess of the value given by Euler's formula. This excess is so marked that even these few tests may be taken to indicate that for computing the safety of I-beam webs against buckling the method common in texts on mechanics of materials gives results which are on the side of safety.

From the twelfth line of Table 8 it will be seen that in all beams but one the fiber stress in shear at mid-web was not much below the yield-point in shear for structural steel, which averages from 25,000 to 35,000 lb. per sq. in. Of course, even under the most favorable circumstances the web of an I-beam may not be counted on to develop without failure a stress in excess of the yield-point in shear of the web material.

In the fifteenth line of Table 8 is given the computed fiber stress in compression developed at the roots of the flange. This fiber stress is computed from equation (7). In the sixteenth line of the table is given the yield-point strength of the material of the beams at the root of the flange.

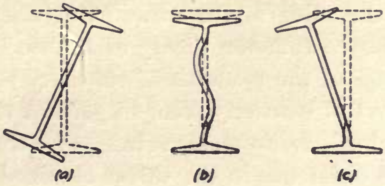

FIG. 8. SHAPES ASSUMED BY I-BEAMS AFTER WEB FAILURE.

This was determined by means of tests of specimens cut from the beams. The compressive fiber stress developed was in all cases not much greater than this yield-point strength of material. However, in all the tests for web failure made at the University of Illinois, before final failure occurred evident signs of structural injury, scaling, etc., had appeared. It is unwise to regard the ultimate compressive fiber stress in the web adjacent to a bearing block as higher than the yield-point strength of the material at the root of the flange. Moreover, the fact should be borne in mind that the material at the root of the flange of an I-beam has a yield-point strength somewhat lower than the material in the flange or in the web. In the absence of special tests the yield-point strength of the structural steel at the root of the flange of an I-beam may be taken as about 30,000

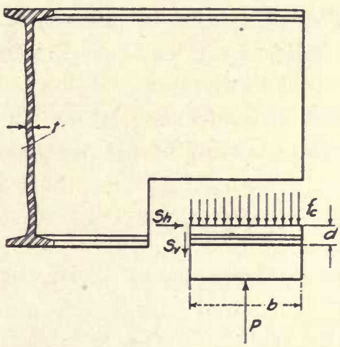

FIG. 9. DIAGRAM OF COMPRESSIVE STRESS IN WEB OF I-BEAM OVER A BEARING BLOCK.

lb. per sq. in., which is near the value obtained in various tests at Illinois and by Marburg and by Hancock. In view of the small amount of data of failure of I-beams by buckling of web, the conclusions given should be regarded as tentative.

15. *Stiffness of I-beams.*—In most of the tests of I-beams referred

to in this bulletin the value of the modulus of elasticity based on the beam deflections and the common theory of flexure is reported. Tables 2, 4 and 5 give values of the modulus of elasticity so computed. These values of the modulus of elasticity seem in general to be lower than the values usually obtained from tension tests of structural steel or of wrought iron. This difference in the values obtained in the two ways is confirmed by Marburg's tests. By means of extensometer tests of samples of material cut from I-beams, Marburg determined the modulus of elasticity of the beam material, and also computed the modulus of elasticity of the beam from load-deflection curves. Table 9 summarizes the average values obtained from the published data of Marburg's tests. It will be seen that the modulus of elasticity obtained from beam deflections is about 10 per cent less than the modulus of elasticity obtained from tension tests of samples of beam material; or in other words, the stiffness of the I-beams was about 10 per cent less than that indicated by tension tests of material.

16. *Summary.*—The following summary is given:

1. The yield-point strength, not the ultimate tensile strength, should be regarded as the ultimate fiber stress for structural steel in flexure.

2. The yield-point strength of structural steel in compression is about the same as the yield-point strength in tension.

3. The slight inelastic action which may be observed in steel I-beams under stresses as low as those used in practice is in general local in its effects and does not indicate the load-carrying capacity of the I-beam, if the load is not reversed in direction.

4. The computed *ultimate* fiber stress for steel I-beams not restrained against sidewise buckling of the compression flange is given by the formula

$$f_1 = 40{,}000 - 60 \frac{ml}{r'}$$

in which f_1 is the extreme fiber stress, in pounds per square inch, computed by the usual flexure formula, l is the length of span of beam in inches, r' is the radius of gyration of the I-section about a gravity axis parallel to the web, and m is a coefficient dependent on the method of loading, ml being a so-called equivalent column length. Values of m for various loadings are given in Table 4. In no case should f_1 be taken greater than the yield-point strength of the material in the flanges. It should be borne in mind that f_1 of this formula is an ultimate, not a working value.

5. A light system of sidewise bracing may so strengthen an I-beam that the full yield-point strength of the material will be developed before

TABLE 8.

WEB FAILURE OF I-BEAMS.

1.	Beam	12-in., 31.5-lb. I-beam	12-in., 31.5-lb. I-beam. Web planed thin	12-in., 31.5-lb. I-beam. Web planed thin	12-in., 31.5-lb. I-beam. Web planed thin	Beth'm Girder Beam 30-in., 175-lb.	20-in. Special* Built-up Girder
2.	Span, ft.	2.92	3.00	3.00	3.00	20	15
3.	Loading	Two points 4⅞ in. each side of center	Two points 4⅞ in. each side of center	Two points 4⅞ in. each side of center	Two points 4⅞ in. each side of center	Quarter points	One load at quarter-point
4.	Material	Steel	Steel	Steel	Steel	Steel	Steel
5.	Number tested	2	1	1	1	3	1
6.	Tested by	Univ. of Illinois	Univ. of Illinois	Univ. of Illinois	Univ. of Illinois	Marburg (Univ. of Pa.)	Turneaure (Univ. of Wis.)
7.	Fiber stress due to direct flexure, lb. per sq. in..................	33,500	28,200	19,300	12,700	31,000	Sidewise buckling prevented by bracing
8.	Vertical distance between flanges, inches (h)	10.52	10.52	10.52	10.52	26.8	14
9.	Thickness of web, inches (t)....	0.35	0.28	0.19	0.16	0.69	0.14
10.	Slenderness ratio for web....... $\frac{l}{r}=\sqrt{12}\,\frac{h \sec 45^\circ}{t}$	148	184	272	324	190	490
11.	Load at failure, pounds.........	190,100	160,500	109,600	72,100	538,400	106,000 (Approx.)
12.	Computed fiber stress (shear and also compression) at middle of web, lb. per sq. in.	25,800	27,200	27,400	21,400	14,800	26,500
13.	$\frac{4\pi^2 E}{(\frac{l}{r})^2}$	53,900	34,900	16,000	11,300	32,700	4,900
14.	Length of block under support, inches	6	6	6	6	12	....
15.	Computed fiber stress (compression) in web adjacent to support, lb. per sq. in..........	45,300	47,800	48,200	37,600	32,500	Stiffeners used at ends and under load
16.	Yield-point strength of material at root of flange, lb. per sq. in....	31,700	33,100	34,000	32,200	28,200	37,700

*Test reported in full in the Journal of the Western Society of Engineers for 1907, p. 788. Failure by sidewise buckling was prevented by bracing. The load at failure is given by Dean Turneaure as that at which very great distortion had taken place and noticeable buckling in the web occurred. Excessive compressive stress in the web adjacent to reactions and concentrated loads was prevented by using stiffeners, well fitted to the flanges.

TABLE 9.

MODULUS OF ELASTICITY OF I-BEAMS AND OF I-BEAM MATERIAL.

Values from results of tests by Marburg at the University of Pennsylvania.

Item	Standard I-beams	Bethlehem I-beams	Bethlehem Girder Beams
Average modulus of elasticity of tension test pieces cut from web, flange and root of flange of I-beam, lb. per sq. in. .... (A)	29,500,000	28,810,000	29,660,000
Average modulus of elasticity of beams determined from deflections, lb. per sq. in. .... (B)	26,300,000	26,570,000	26,120,000
(B) : (A).................................	0.892	0.937	0.882

failure occurs, but a stiff bracing capable of resisting sidewise bending moment is necessary to prevent sudden failure by sidewise buckling, once the yield-point of the beam flanges is reached. Separators between the webs of I-beams do not furnish a stiff bracing against sidewise buckling.

6. In investigating the safety of an I-beam as regards web failure three possible causes of failure should be considered:

(a) Failure by shearing stress in the web. The yield-point strength of structural steel in shear should be regarded as the ultimate fiber stress for the web.

(b) Failure by buckling of web. The buckling strength of a strip of web inclined 45 degrees to the flanges as computed by Euler's formula for fixed-ended columns was developed in several tests without collapse of the web.

(c) Failure by compressive stress in the part of the web adjacent to a bearing block. The value of this stress may be roughly estimated from the formula given by Hudson,

$$f_w = \frac{P}{bt}$$

in which f_w is the fiber stress in compression in pounds per square inch, b is the length of bearing block in inches, t is the thickness of web in inches, and P is the concentrated load or the reaction in pounds. The yield-point strength of the material at the root of the flange of the I-beam should be regarded as the ultimate value for f_w.

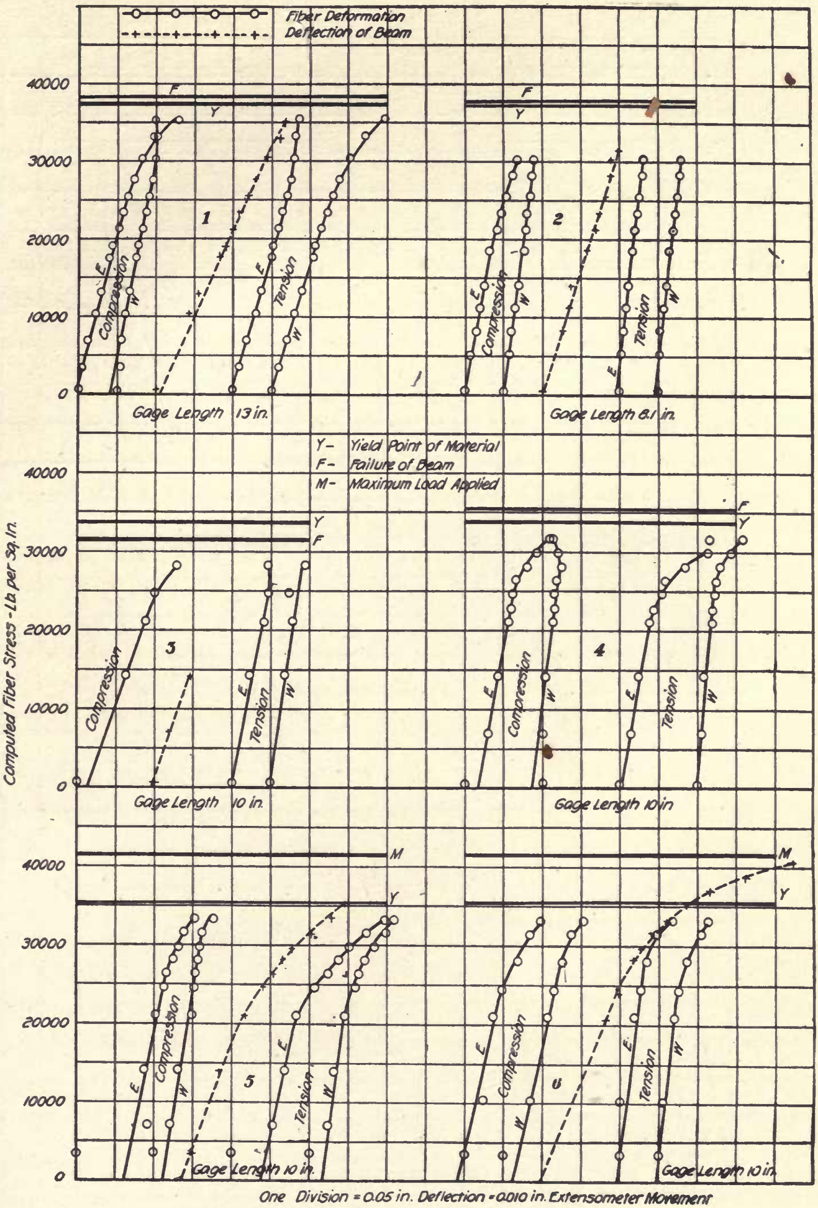

FIG. 10. RESULTS OF TESTS 1-6.

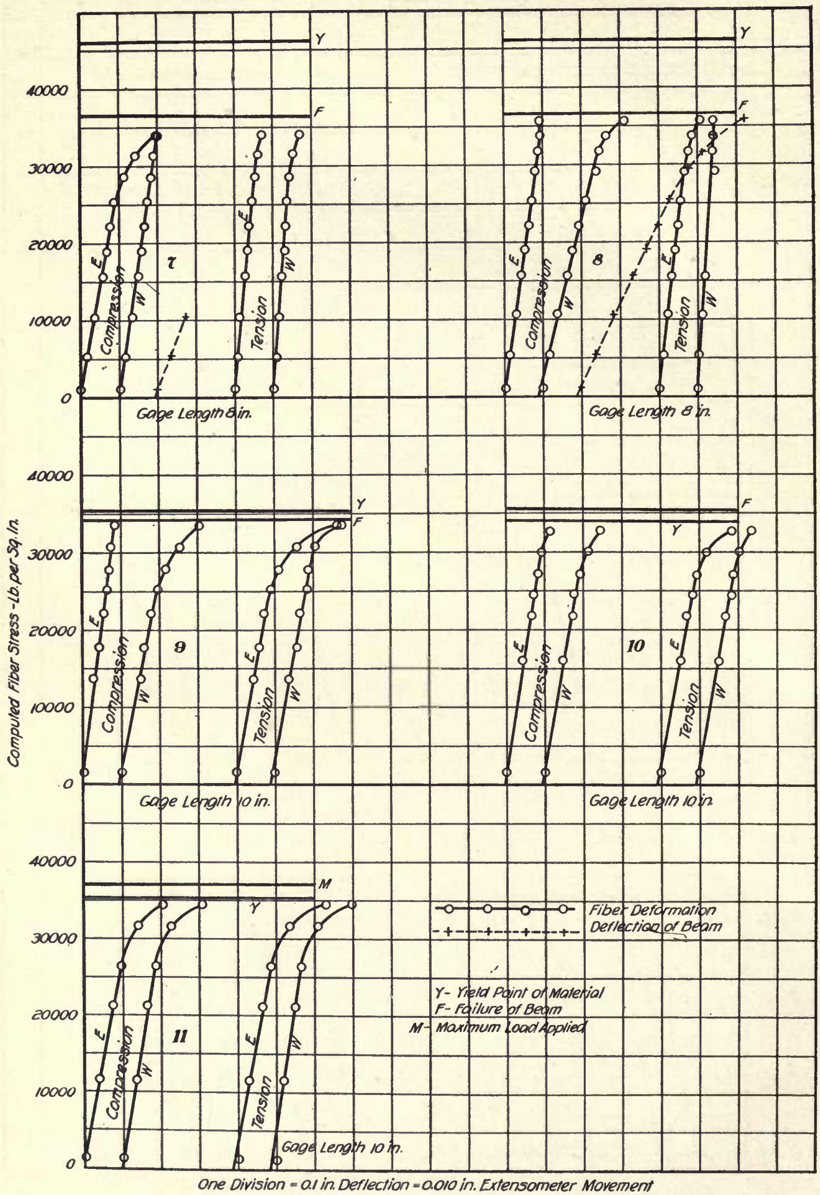

FIG 11. RESULTS OF TESTS 7-11.

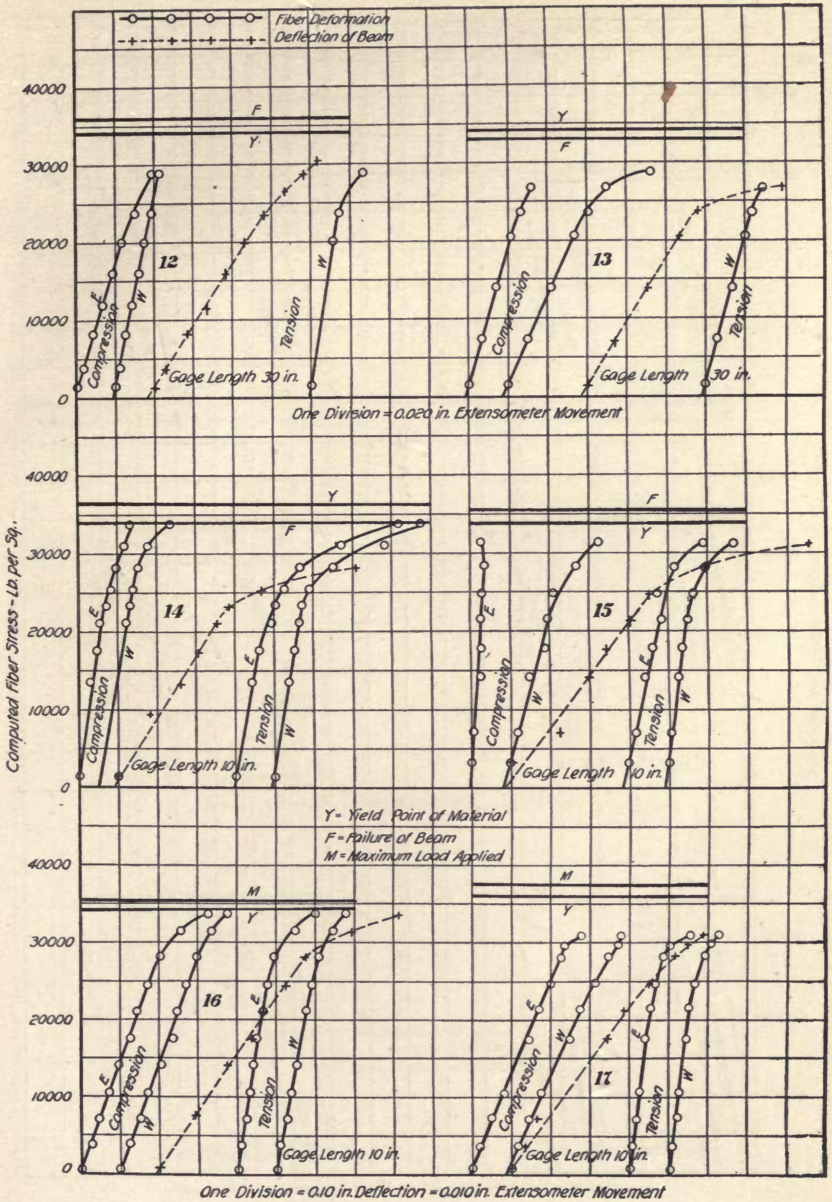

FIG. 12. RESULTS OF TESTS 12-17.

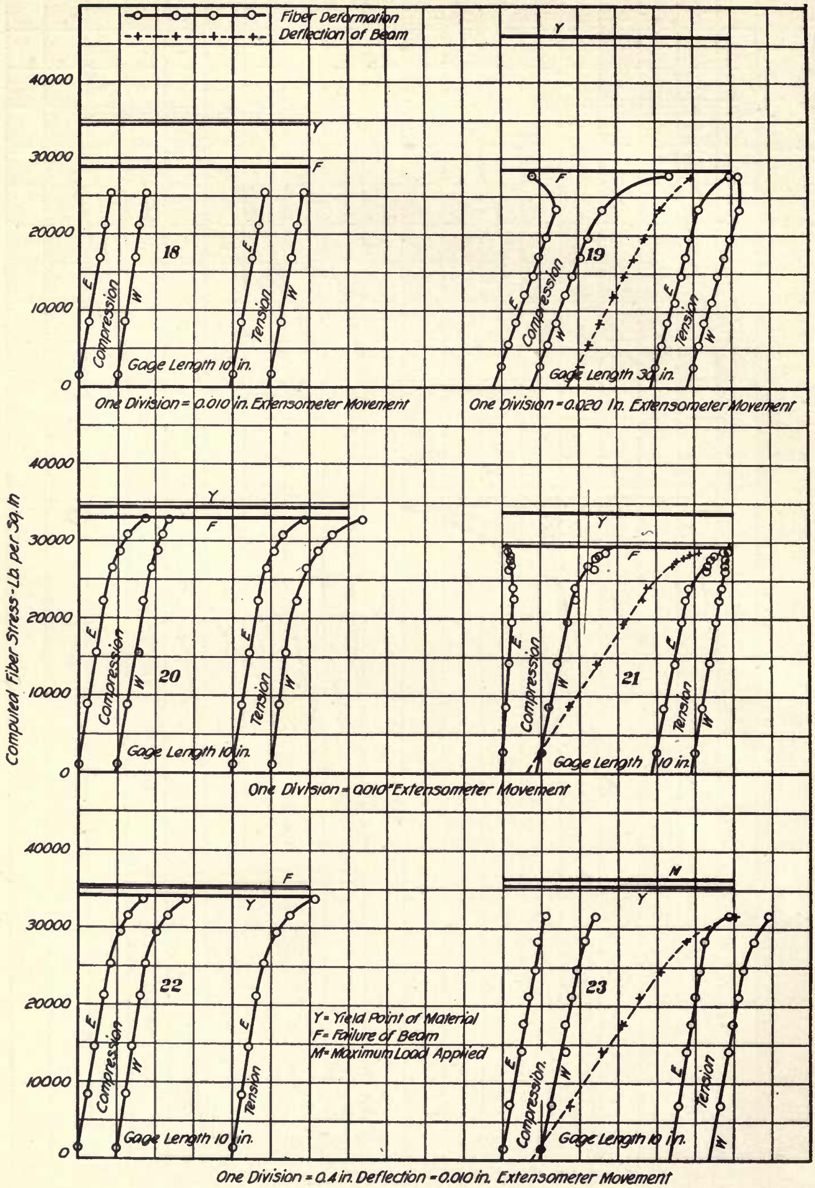

FIG. 13. RESULTS OF TESTS 18-23.

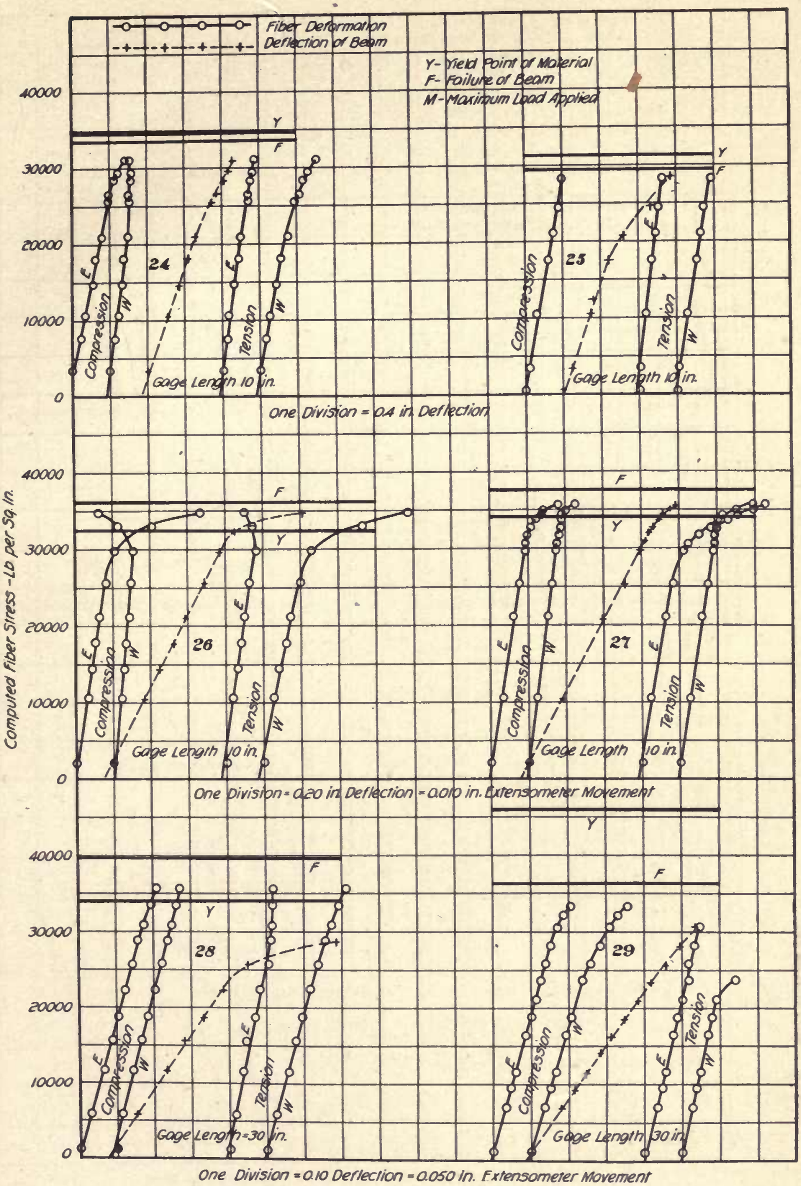

Fig 14. Results of Tests 24-29.

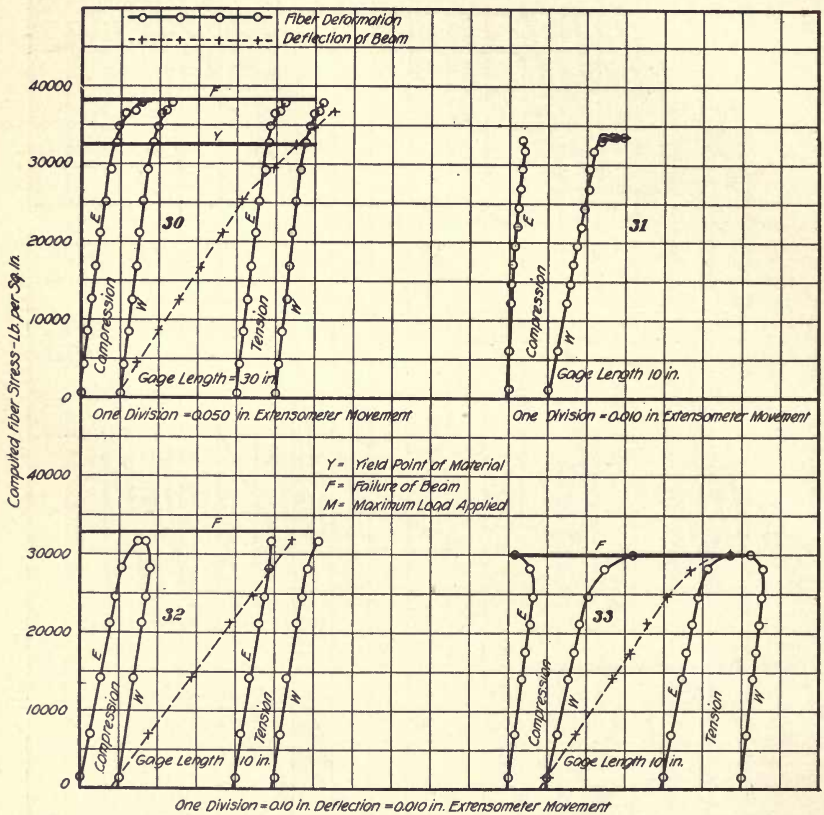

FIG. 15. RESULTS OF TESTS 30-33.

PUBLICATIONS OF THE ENGINEERING EXPERIMENT STATION

Bulletin No. 1. Tests of Reinforced Concrete Beams, by Arthur N. Talbot. 1904. *None available.*

Circular No. 1. High-Speed Tool Steels, by L. P. Breckenridge. 1905. *None Available.*

Bulletin No. 2. Tests of High-Speed Tool Steels on Cast Iron, by L. P. Breckenridge and Henry B. Dirks. 1905. *None available.*

Circular No. 2. Drainage of Earth Roads, by Ira O. Baker. 1906. *None available.*

Circular No. 3. Fuel Tests with Illinois Coal (Compiled from tests made by the Technologic Branch of the U. S. G. S., at the St. Louis, Mo., Fuel Testing Plant, 1904-1907), by L. P. Breckenridge and Paul Diserens. 1909. *Thirty cents.*

Bulletin No. 3. The Engineering Experiment Station of the University of Illinois, by L. P. Breckenridge. 1906. *None available.*

Bulletin No. 4. Tests of Reinforced Concrete Beams, Series of 1905, by Arthur N. Talbot. 1906. *Forty-five cents.*

Bulletin No. 5. Resistance of Tubes to Collapse, by Albert P. Carman and M. L. Carr. 1906. *Fifteen cents.*

Bulletin No. 6. Holding Power of Railroad Spikes, by Roy I. Webber. 1906. *None available.*

Bulletin No. 7. Fuel Tests with Illinois Coals, by L. P. Breckenridge, S. W. Parr, and Henry B. Dirks. 1906. *None available.*

Bulletin No. 8. Tests of Concrete: I. Shear; II. Bond, by Arthur N. Talbot. 1906. *None available.*

Bulletin No. 9. An Extension of the Dewey Decimal System of Classification Applied to the Engineering Industries, by L. P. Breckenridge and G. A. Goodenough. 1906. Revised Edition 1912. *Fifty cents.*

Bulletin No. 10. Tests of Concrete and Reinforced Concrete Columns, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 11. The Effect of Scale on the Transmission of Heat through Locomotive Boiler Tubes, by Edward C. Schmidt and John M. Snodgrass. 1907. *None available.*

Bulletin No. 12. Tests of Reinforced Concrete T-beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 13. An Extension of the Dewey Decimal System of Classification Applied to Architecture and Building, by N. Clifford Ricker. 1907. *Fifty cents.*

Bulletin No. 14. Tests of Reinforced Concrete Beams, Series of 1906, by Arthur N. Talbot. 1907. *None available.*

Bulletin No. 15. How to Burn Illinois Coal without Smoke, by L. P. Breckenridge. 1908. *Twenty-five cents.*

Bulletin No. 16. A Study of Roof Trusses, by N. Clifford Ricker. 1908. *Fifteen cents.*

Bulletin No. 17. The Weathering of Coal, by S. W. Parr, N. D. Hamilton, and W. F. Wheeler. 1908. *None available.*

Bulletin No. 18. The Strength of Chain Links, by G. A. Goodenough and L. E. Moore. 1908. *Forty cents.*

Bulletin No. 19. Comparative Tests of Carbon, Metallized Carbon, and Tantalum Filament Lamps, by T. H. Amrine. 1908. *None available.*

Bulletin No. 20. Tests of Concrete and Reinforced Concrete Columns, Series of 1907, by Arthur N. Talbot. 1908. *None available.*

Bulletin No. 21. Tests of a liquid Air Plant, by C. S. Hudson and C. M. Garland. 1908. *Fifteen cents.*

Bulletin No. 22. Tests of Cast-Iron and Reinforced Concrete Culvert Pipe, by Arthur N. Talbot. 1908. *Thirty-five cents.*

Bulletin No. 23. Voids, Settlement, and Weight of Crushed Stone, by Ira O. Baker. 1908. *Fifteen cents.*

Bulletin No. 24. The Modification of Illinois Coal by Low Temperature Distillation, by S. W. Parr and C. K. Francis. 1908. *Free upon request.*

Bulletin No. 25. Lighting Country Homes by Private Electric Plants, by T. H. Amrine. 1908. *Free upon request.*

Bulletin No. 26. High Steam Pressures in Locomotive Service. A Review of a Report to the Carnegie Institution of Washington, by W. F. M. Goss. 1908. *Free upon request.*

Bulletin No. 27. Tests of Brick Columns and Terra Cotta Block Columns, by Arthur N. Talbot and Duff A. Abrams. 1909. *Thirty cents.*

Bulletin No. 28. A Test of Three Large Reinforced Concrete Beams, by Arthur N. Talbot. 1909. *Free upon request.*

Bulletin No. 29. Tests of Reinforced Concrete Beams: Resistance to Web Stresses, Series of 1907 and 1908, by Arthur N. Talbot. 1909. *Free upon request.*

Bulletin No. 30. On the Rate of Formation of Carbon Monoxide in Gas Producers, by J. K. Clement, L. H. Adams, and C. N. Haskins. 1909. *Free upon request.*

Bulletin No. 31. Fuel Tests with House-Heating Boilers, by J. M. Snodgrass. 1909. *Free upon request.*

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Bulletin No. 33. Tests of Tungsten Lamps, by T. H. Amrine and A. Guell. 1909. *Twenty cents.*

Bulletin No. 34. Tests of Two Types of Tile Roof Furnaces under a Water-Tube Boiler, by J. M. Snodgrass. 1909. *Free upon request.*

Bulletin No. 35. A Study of Base and Bearing Plates for Columns and Beams, by N. Clifford Ricker. 1909. *Twenty cents.*

Bulletin No. 36. The Thermal Conductivity of Fire-Clay at High Temperatures, by J. K. Clement and W. L. Egy. 1909. *Free upon request.*

Bulletin No. 37. Unit Coal and the Composition of Coal Ash, by S. W. Parr and W. F. Wheeler. 1909. *Thirty-five cents.*

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Bulletin No. 39. Tests of Washed Grades of Illinois Coal, by C. S. McGovney. 1909. *Free upon request.*

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Bulletin. No. 43. Freight Train Resistance, by Edward C. Schmidt. 1910. *Seventy-five cents.*

Bulletin No. 44. An investigation of Built-up Columns under Load, by Arthur N. Talbot and Herbert F. Moore. 1911. *Free upon request.*

Bulletin No. 45. The Strength of Oxyacetylene Welds in Steel, by Herbert L. Whittemore. 1911. *Free upon request.*

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Bulletin No. 50. Tests of a Suction Gas Producer, by C. M. Garland and A. P. Kratz. 1912. *Free upon request.*

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Bulletin No. 52. An Investigation of the Strength of Rolled Zinc, by Herbert F. Moore. 1912. *Free upon request.*

Bulletin No. 53. Inductance of Coils, by Morgan Brooks and H. M. Turner. 1912. *Free upon request.*

Bulletin No. 54. Mechanical Stresses in Transmission Lines, by A. Guell. 1912. *Free upon request.*

Bulletin No. 55. Starting Currents of Transformers, with Special Reference to Transformers with Silicon Steel Cores, by Trygve D. Yensen. 1912. *Free upon request.*

Bulletin No. 56. Tests of Columns: An Investigation of the Value of Concrete as Reinforcement for Structural Steel Columns, by Arthur N. Talbot and Arthur R. Lord. 1912. *Free upon request.*

Bulletin No. 57. Superheated Steam in Locomotive Service. A Review of Publication No. 127 of the Carnegie Institution of Washington, by W. F. M. Goss. 1912. *Free upon request.*

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Bulletin No. 61. Characteristics and Limitations of the Series Transformer, by A. R. Anderson and H. R. Woodrow. 1913. *Free upon request.*

Bulletin No. 62. The Electron Theory of Magnetism, by Elmer H. Williams. 1913. *Free upon request.*

Bulletin No. 63. Entropy-Temperature and Transmission Diagrams for Air, by C. R. Richards. 1913. *Free upon request.*

Bulletin No. 64. Tests of Reinforced Concrete Buildings under Load, by Arthur N. Talbot and Willis A. Slater. 1913. *Free upon request.*

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